Solving Convolutional LASSO with LCA

Neuromorphic Computing Lab | Intel Labs

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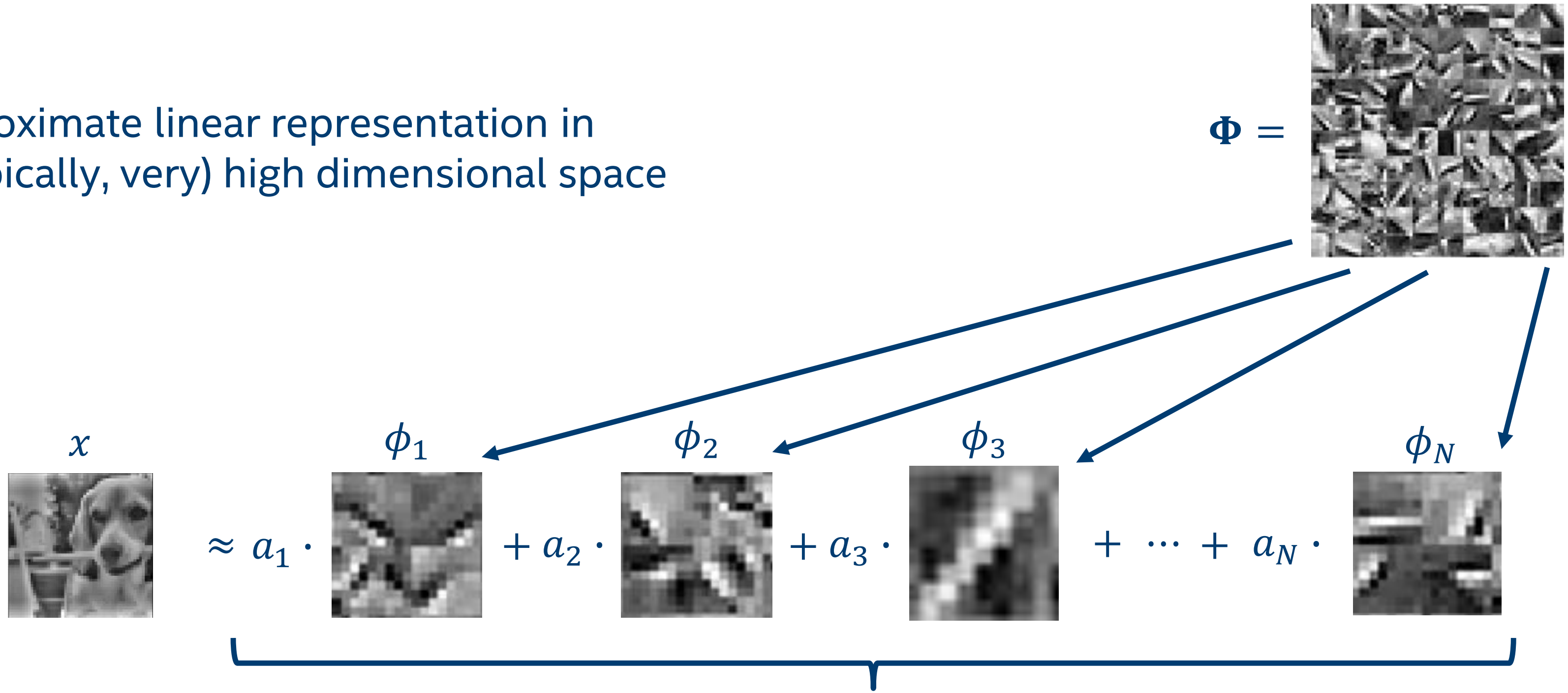
SPARSE CODING AS A LASSO PROBLEM

Sparse coding 2D images

Approximate linear representation in
a (typically, very) high dimensional space



Sparse → most of the a_js are 0

Conventionally expensive but solved efficiently on Loihi!

Sparse coding: LASSO formulation

Reconstruction error

ℓ-1 regularization term

LASSO Cost function: $E = \frac{1}{2}\|\mathbf{x} - \boldsymbol{\Phi} \cdot \boldsymbol{a}\|_2^2 + \lambda \cdot \|\boldsymbol{a}\|_1$

sparse code: $\boldsymbol{a}^* = \underset{\boldsymbol{a}}{\operatorname{argmin}} E(\boldsymbol{a})$

$\|\boldsymbol{a}\|_1 = \sum_j |a_j|$

$\boldsymbol{\Phi} \cdot \boldsymbol{a} = \sum_k \Phi_{jk} a_k$

approx. reconstruction

LASSO to LCA

- Want to solve LASSO…
- Using conventional gradient descent leads to ISTA/FISTA solvers.
- From ISTA derive LCA (locally competitive algorithm) dynamics.

LCA* solves LASSO by gradient descent

(*) C. J. Rozell, D. H. Johnson, and R. G. Baranuik *Neural Computation*, 20:2526 – 2563, 2008.

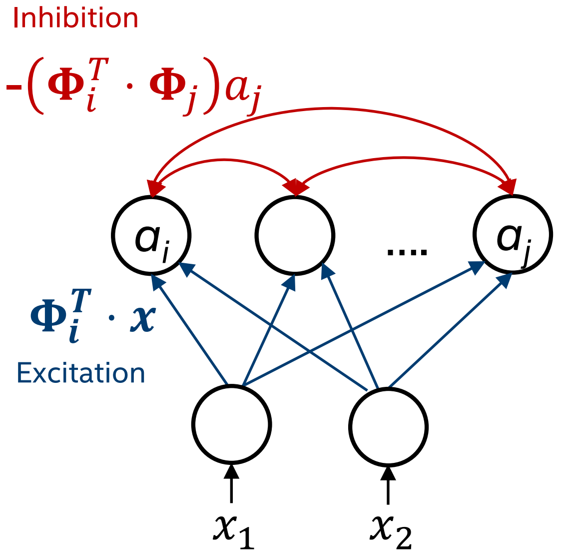
LCA-derived network structure

LCA dynamics:

$$\dot{\boldsymbol{u}} = \frac{1}{\tau}(\boldsymbol{\Phi}^T\boldsymbol{x} - \boldsymbol{u} - (\boldsymbol{\Phi}^T\boldsymbol{\Phi} - \boldsymbol{I}) \cdot \boldsymbol{a})$$

$$\boldsymbol{a} = \mathcal{T}_\lambda(\boldsymbol{u})$$

Network structure:



Feature neurons compete to represent to reconstruct inputs

Analog to Spiking LCA: Rate Coding

- Equivalence between analog LCA dynamics and spike based models*
  - Sparse code a is represented by neuron spiking rates

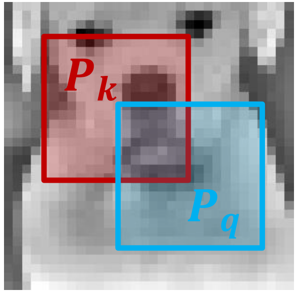
Analog-LCA ⟺ Spiking-LCA

* P. Tang, T.-H. Lin, and M. Davies, arXiv:1705.05475v1

Resource efficiency through Convolutional LCA

Image: $\boldsymbol{x} \in \mathbb{Z}^{n \times n}$

Image patches: $\boldsymbol{P}_k, \boldsymbol{P}_q \in \{0,1\}^{n \times n}, |P_k| = r$

$\boldsymbol{x} =$ 

Linearized image: $\boldsymbol{x}' \in \mathbb{Z}^{n^2} = \mathbb{Z}^m$

Elementary dictionary: $\boldsymbol{\Psi} \in \mathbb{Z}^{r \times p}$

$\boldsymbol{\Psi} =$ 

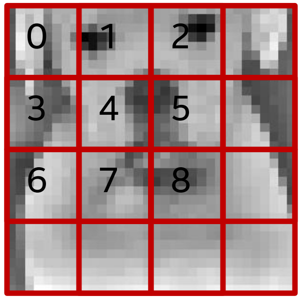
Patch dictionary:
$\boldsymbol{\Phi}_k \in \mathbb{Z}^{m \times p}, \boldsymbol{\Phi}_\mathrm{k}[\boldsymbol{P}_k, :] = \boldsymbol{\Psi}$

- Extend to convolutional LCA
  - Image patches
  - Generalized (inhibitory) interaction matrix:
    - $\mathbf{W}_{\mathrm{k,q}} = \Phi_\mathrm{k}^T \cdot \Phi_\mathrm{q} - I$
  - Neurons per patch integrate input from overlapping patches
  - Significant increase in number of connections

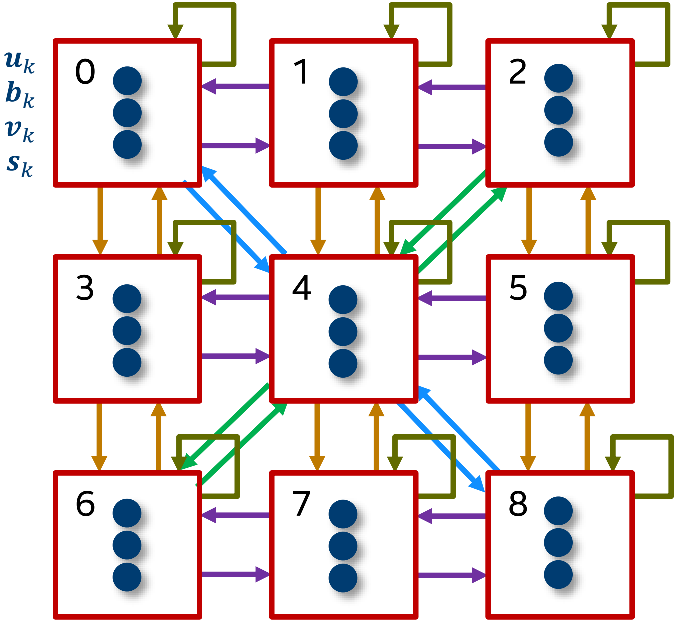
Loihi takes advantage of convolutional topology for efficient network compression

Neural network structure

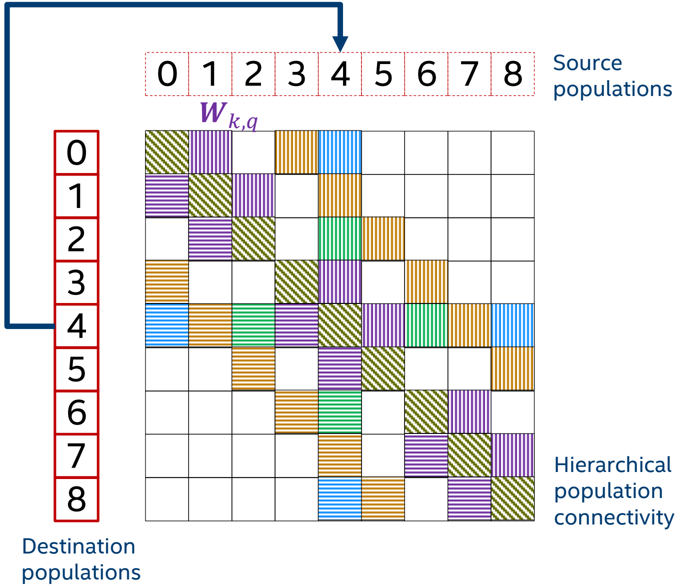
Input image with patches



Patches -> Populations with population connectivity



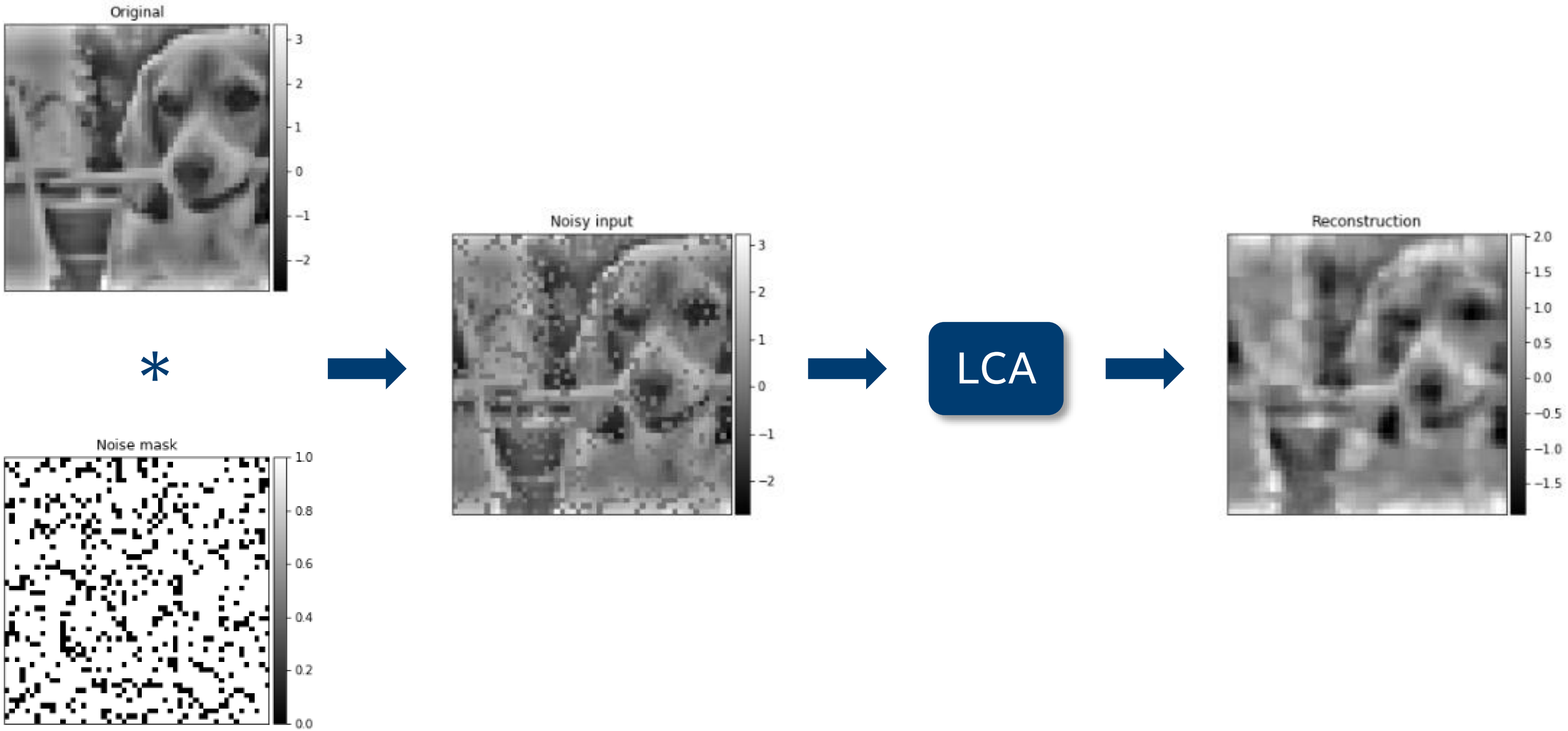
Connectivity matrix



Sharing of same sub-matrices reduces resource requirements!

TUTORIAL: IMAGE DE-NOISING WITH LCA ON LOIHI

De-noising workload



Tutorial: LCA Module for Image De-Noising

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* Other names and brands may be claimed as the property of others

Questions?